(Following Paper ID and Roll No. to be filled in your Answer Book)

PAPER ID : 9601 Roll No. | | | | | | | | | | |

B. Tech.

(Semester-I) Theory Examination, 2012-13

MATHEMATICS–I

Time : 3 Hours] *[Total Marks : 100*

Note : Attempt questions from each Section as per instructions. The symbols have their usual meaning.

Section-A

Attempt *all* parts of this question. Each part carries 2 marks. 2×10=20

1. (a) Find y_n, if $y = \dfrac{ax+b}{cx+d}$.

(b) Find all the asymptotes of the curve $xy^2 = 4a^2(2a-x)$.

(c) If $z = xyf\left(\frac{x}{y}\right)$, show that :

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 2z.$$

(d) Determine the point(s) where the function $u = x^2 + y^2 + 6x + 12$ has a maximum or minimum.

(e) Evaluate $\dfrac{\Gamma\left(\frac{8}{3}\right)}{\Gamma\left(\frac{2}{3}\right)}$.

(f) Change the order of integration :

$$\int_0^a \int_0^{2\sqrt{ay}} f(x, y)\,dx\,dy + \int_0^{3a} \int_0^{3a-y} f(x, y)\,dx\,dy$$

(g) If $\vec{A}$ and $\vec{B}$ are irrotational, prove that $\vec{A} \times \vec{B}$ is solenoidal.

(h) Find $\int_C \hat{t}.d\vec{r}$, where $\hat{t}$ is the unit tangent vector and C is the unit circle in the xy-plane about the origin.

(i) If A is a skew-Hermitian matrix, prove that (iA) is Hermitian matrix.

(j) Find the sum and product of eigenvalues of the matrix :

$$\begin{bmatrix} 1 & 6 & 1 \\ 1 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

Section-B

Attempt any *three* parts of this question. Each part carries 10 marks. $10\times3=30$

2. (a) Find y_3 when $y=\sqrt{1+x^2}\,.\sin x$ by Leibnitz theorem.

(b) If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, then prove that :

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0.$$

(c) Prove that :

$$\iiint\frac{dx\,dy\,dz}{\sqrt{a^2-x^2-y^2-z^2}}=\frac{\pi^2a^2}{8},$$

the integral being extended for all positive values of the variables for which the expression is real.

(d) Apply Stoke's theorem to evaluate :

$$\oint_C [(x+y)\,dx + (2x-z)\,dy + (y+z)dz],$$

where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6).

(e) Find the square matrix A whose eigenvalues are 1, 2 and 3 and their corresponding eigenvectors are $[1, 0, -1]^t, [0, 1, 0]^t$ and $[1, 0, 1]^t$ respectively.

Section-C

Attempt *all* questions of this Section. Attempt any *two* parts from each question. Each question carries 10 marks. $10\times5=50$

3. (a) If $y = x^n \ln x$, prove that $xy_{n+1} = n!$.

(b) Prove that :

$$f\left(\frac{x^2}{1+x}\right) = f(x) - \frac{x}{1+x} f'(x) + \frac{1}{2!}\frac{x^2}{(1+x)^2} f''(x) - \ldots$$

(c) Trace the curve $y^2(a-x) = x^2(a+x)$.

4. (a) If $u = x_1 + x_2 + x_3 + x_4$, $uv = x_2 + x_3 + x_4$, $uvw = x_3 + x_4$ and $uvwt = x_4$, find :

$$\frac{\partial(x_1, x_2, x_3, x_4)}{\partial(u, v, w, t)}.$$

(b) What error in the common logarithm of a number will be produced by an error of 1% in the number ?

(c) If $x^x . y^y . z^z = C$, show at $x = y = z$:

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{-1}{x \ln(ex)}.$$

5. (a) Evaluate the integral :

$$\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} \, dy \, dx.$$

(b) Find, by double integration, the area of the region enclosed by the curves $x^2 + y^2 = a^2$, $x + y = a$ in the first quadrant.

(c) Show that :

$$\int_0^1 \frac{dx}{\sqrt{1-x^n}} = \frac{\sqrt{\pi}.\sqrt{\left(\frac{1}{n}\right)}}{n.\sqrt{\left(\frac{1}{n}+\frac{1}{2}\right)}}.$$

6. (a) If $\phi(x, y) = \frac{1}{2}\ln(x^2 + y^2)$, show that :

$$\text{grad } \phi = \frac{\vec{r} - (\hat{k}.\vec{r})\hat{k}}{\{\vec{r} - (\hat{k}.\vec{r})\hat{k}\}.\{\vec{r} - (\hat{k}.\vec{r})\hat{k}\}}.$$

(b) If $u = x + y + z$, $v = x^2 + y^2 + z^2$ and $w = xy + yz + zx$, show that grad u, grad v and grad w are coplanar.

(c) Consider a vector field :

$$\vec{F} = (x^2 - y^2 + x)\hat{i} - (2xy + y)\hat{j}.$$

Show that the field is irrotational and find its scalar potential. Hence evaluate the line integral from (1, 2) to (2, 1).

7. (a) Find the inverse of the matrix :

$$\begin{bmatrix} i & -1 & 2i \\ 2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$

by employing elementary transformations.

(b) Find the value of λ such that the following equations have unique solution :

$$\lambda x + 2y - 2z = 1$$

$$4x + 2\lambda y - z = 2$$

$$6x + 6y + \lambda z = 3.$$

(c) Examine the linear dependence of the vectors $[1, -1, 1]$, $[2, 1, 1]$ and $[3, 0, 2]$. If dependent, find the relation between them.